PROJECTION. (Contnd.)

When I started thiking about projection, it feels simple

⇒ A shadow of something over sub-space

Subspace can be line, plane etc.

But,

$r \in U^{\perp}$ is perpendicular to all of U

But,

Can a point in 2-D be $\perp^{ar}$ to all points in circle?

Genuine Question ???

⇒ {PROJECTION IS FOR LINEAR ALGEBRA}

$\perp$ar means vector are $\perp$ar, not lines

Go back if you donot understand this line.

$x^2 + y^2 = 1$

$\Downarrow$

equation of circle doesnot follows algebra sum / subtract etc.

Hence

$$x^2 + y^2 = 1 \notin U$$

It cannot be a space. All formulaes which we have study so far cannot apply to any random space

Space can be $\Rightarrow$

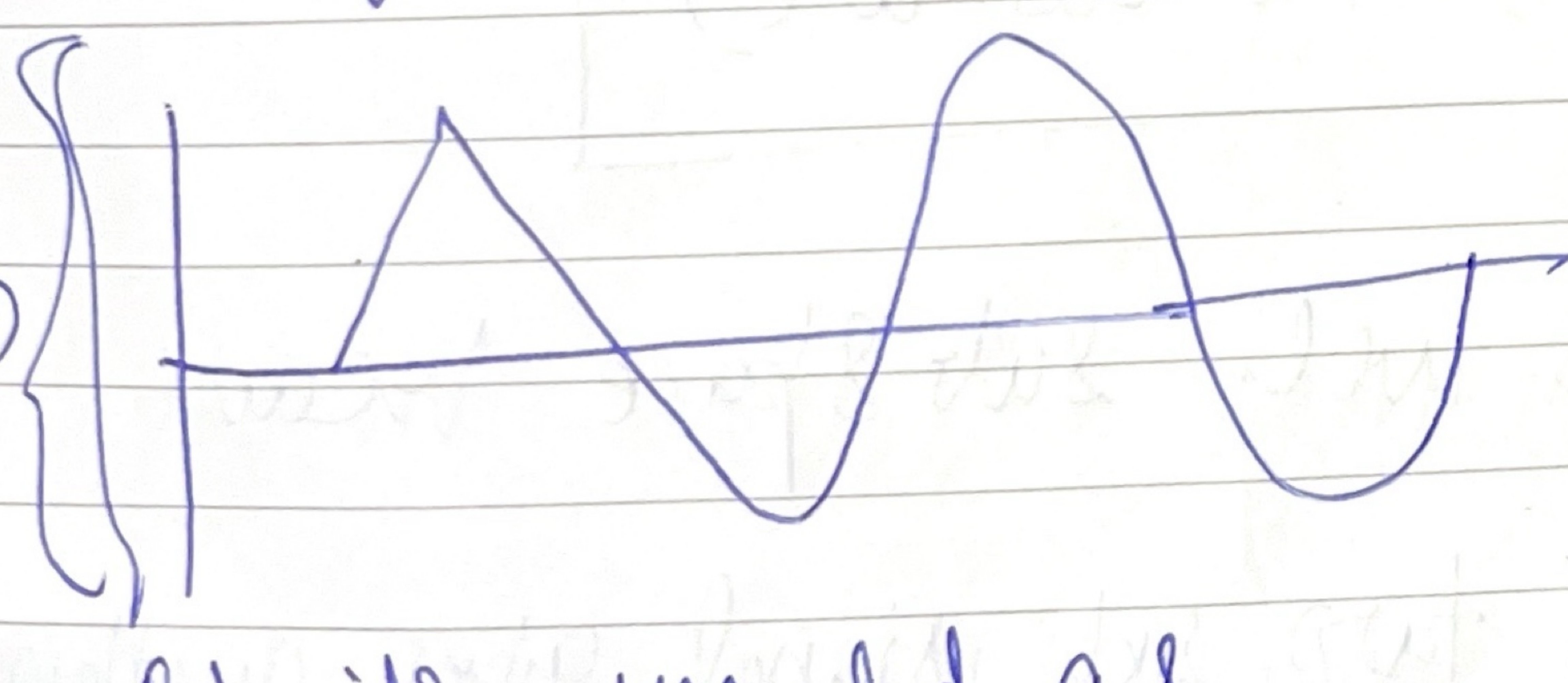
- line
- plane
- Complete space

But anything that follow algebra rules.

⇒ That how equation work.

HOW TO PROJECT SOUND WAVE?

- how to remove noise.
- does sound wave follow linear algebra.

⇒ 

Sound wave

But its recorded as

$$y = \begin{bmatrix} 0.2 \\ 0.7 \\ -0.1 \\ -0.8 \\ 0.1 \end{bmatrix} \begin{matrix} \text{up} \\ \text{down} \\ \text{up} \end{matrix}$$

Imp

©Smiley

So sound wave is converted into a vector such as this, belongs to

R^5, $\underbrace{R^{1000}}_{\text{as per sampling}}$

$\Rightarrow V = \text{span}\{\sin(\omega t), \cos(\omega t)\}$

$\Downarrow$

This means every allowed signal is

$$\boxed{a \sin(\omega t) + b \cos(\omega t)}$$

$\Downarrow$

This is as well sub space because

- adding two such signal gives another one of same form
- scaling one keeps form same.

⇒ A curve can still come from a linear model

Linear ≠ Straight Line

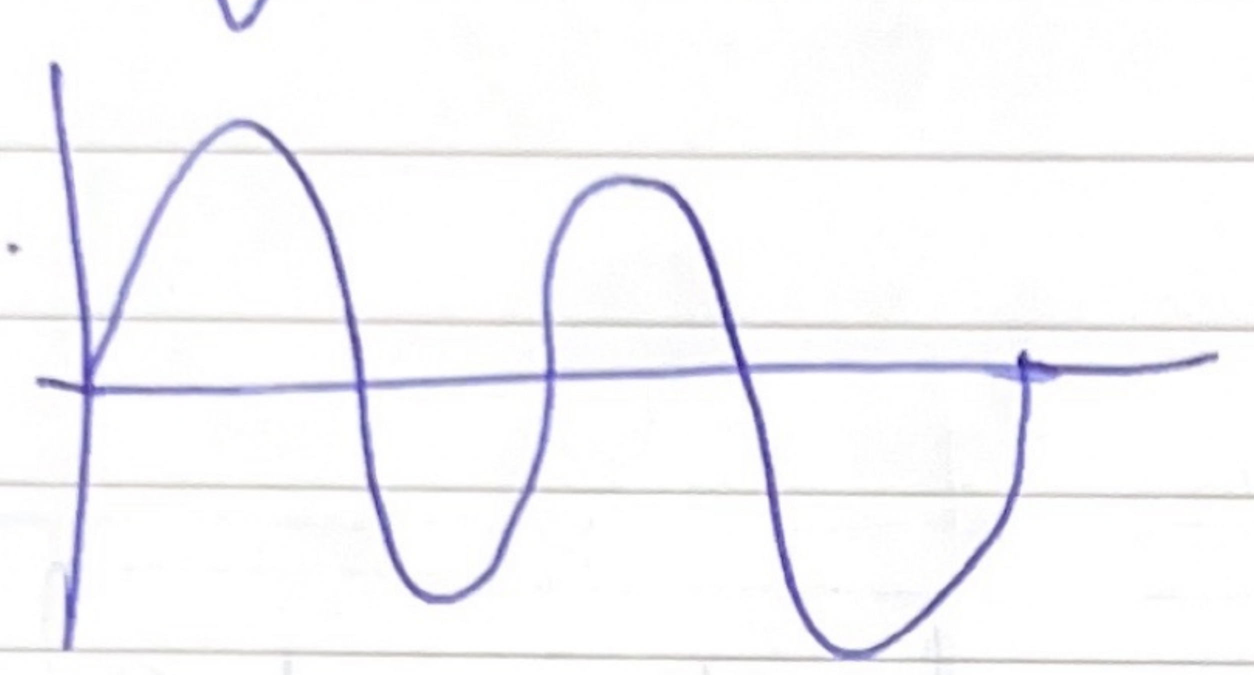
eg $a \sin(t) + b \cos(t)$

Why different voices sound different?

Imp

- Pitch (how fast wave form repeats)
- Loudness (magnitude of wave)
- tone/timbre (shape of wave)
- shape of vocal tract
- speed of change over time

$\boxed{\text{How sound waves form sound/words?}}$



(Its a wave)

How it store Words?

Think, these numbers, graphs hit ear drum and ear form sound & brain interpret it word

MAGIC

↓

This is how God built us. This even prooves we are made.

i. e. Some natural power exists. ....

GOD

e.g. Let y be voice sample :-

$$y = \begin{bmatrix} 1.0 \\ 2.2 \\ 2.9 \\ 1.1 \end{bmatrix} \quad \text{Let, } x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

So,

$$X = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$ Imp

(This is choosen by us. How? Hardest part)

Now we solve,

$$\min_{\theta} \|X\theta - y\|^2$$

$$\theta = \begin{bmatrix} 0.67 \\ 1.07 \end{bmatrix}$$ how?

Let, $\theta = \begin{bmatrix} a \\ b \end{bmatrix}$

Then,

$$\min_{a,b} \left\| \begin{bmatrix} a \\ 2a+b \\ a+2b \\ b \end{bmatrix} - \begin{bmatrix} 1.0 \\ 2.2 \\ 2.9 \\ 1.1 \end{bmatrix} \right\|^2$$

$$\min_{a,b} \left((a-1.0)^2 + (2a+b-2.2)^2 + (a+2b-2.9) \right)^2$$

$\Rightarrow$ Cannod differenciate as we have 2 variables

$$\boxed{\text{Now, } \quad X^T X Q = X^T y}$$ $\Rightarrow$ need another equation which allow reduction.

$$X^T = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

Step B, Compute $X^T X$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix}$$

$1^2 + 2^2 + 1^2 + 0^2 =$

$$\dot{X}^T y = \begin{bmatrix} 8.3 \\ 9.1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1.0 \\ 2.2 \\ 2.9 \\ 1.1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 8.3 \\ 9.1 \end{bmatrix}$$

This gives system

$$\left\{ \begin{array}{l} 4a + 4b = 8.3 \\ 4a + 6b = 9.1 \end{array} \right\}$$

Now you have 2 equations to solve

Sol

$$\theta = \begin{bmatrix} 0.67 \\ 1.07 \end{bmatrix}$$

$$\hat{y} = X\theta = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.67 \\ 1.07 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} 0.67 \\ 2.41 \\ 2.81 \\ 1.07 \end{bmatrix} \Longleftarrow 1$$

$$r = \begin{bmatrix} 1.0 \\ 2.2 \\ 2.9 \\ 1.1 \end{bmatrix} - \begin{bmatrix} 0.67 \\ 2.41 \\ 2.81 \\ 1.07 \end{bmatrix} = \begin{bmatrix} 0.33 \\ -0.21 \\ 0.09 \\ 0.03 \end{bmatrix} - 2$$

Check if (home-work for you)

$$\boxed{X^T r = \begin{bmatrix} 0 \\ 0 \end{bmatrix}} - 3$$

How,

$$\min_{\theta} \|X\theta - y\|^2$$

At optimum

$$r = y - X\theta$$

and

$$X^T r = 0$$

So,

$$X^T (y - X\theta) = 0$$

Thus,

$$X^T X \theta = X^T y$$

If invertible,

$\Rightarrow$ Put 1 in 2 to get result

$$\boxed{\theta = (X^T X)^{-1} X^T y} \quad - 1$$

Then fitted point,

$$\Rightarrow \boxed{\hat{y} = X\theta} \quad - 2$$

&

$$\Rightarrow \boxed{X^T X \theta = X^T y} \quad - 3$$

$$\Rightarrow \boxed{\hat{y} = Py} \quad - 5$$

All five brings, result

$$\boxed{\hat{y} = Py, \quad P = X(X^T X)^{-1} X^T}$$

$\Downarrow$

P = Projection matrix

⇒ Enjoy, Now find

Projection in any **dimention.**